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ON THE DESIGN OF STRUCTURAL MODELS TO STUDY THE THERMAL STRESS PHENOMENON

By

L. ALBERT SCIPIO II

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By

L. ALBERT SCIPIO II

Department of Aeronautical Engineering, University of Minnesota, Minneapolis, Minn.

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***Accorded the D. B. Steinman Award for Research in Structural Engineering in 1958 by The New York Academy of Sciences**

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ON THE DESIGN OF STRUCTURAL MODELS TO STUDY THE THERMAL STRESS PHENOMENON*

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Department of Aeronautical Engineering
University of Minnesota, Minneapolis, Minn.

INTRODUCTION

Elastic or structural similarity is generally very simple to formulate. However, the complexities associated with thermal stress, buckling and related behavior, plastic flow, use of material for the model different from that in the prototype, and problems associated with fluid-structural combinations such as flutter, require very comprehensive analysis.

A true structural model gives an accurate prediction of distortion as well as stress resulting from all four basic types of loadings: (1) axial, (2) shear, (3) torsion, and (4) bending. The structural characteristics of the prototype such as deflection, strains, stresses, buckling loads, and/or combinations thereof, could be obtained from a model that is geometrically similar, if all dimensions, connections, spacings, and so on have been reproduced to scale.

In order to maintain elastic similarity, it is necessary that the mean deflection of the model and that of the prototype be similar and of the same scale N_L.† In addition, it is required that the model simulate the buckling characteristic of the prototype and, in the case of wing structures, the local buckling of the skin.

Elastic similarity is discussed from the standpoint of two structural types: (1) solid and (2) wing-type structures.

The following four basic assumptions are made: (1) the stress field is a homogeneous, isotropic, elastic medium, and creep is not present; (2) the elastic proportional limits of the material of the model or the prototype are not exceeded; (3) the stress fields are slowly varying functions of time; and (4) the physical properties of the material are not independent of temperature.

The symbols used in this presentation are listed in TABLES 1 and 2.

---

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†Characteristic length scale factor = model scale factor.

TABLE 1

ENGINEERING SYMBOLS

d	A characteristic length, thickness	X, Y, Z	Body forces
		x, y, z	Space coordinates
E	Young's modulus of elasticity	α	Coefficient of thermal expansion
e	Component of total deformation		
F	Component of external force	δ	Rib and web spacing
h	Characteristic thickness	ϵ	Component of deformation
N	Scale factor, sectional forces	γ	Component of shear deformation
n	Outward normal to surface of body	κ	Thermal diffusivity
		σ	Stress component
p	Pressure	τ	Shear stress component
u, v, w	Components of deformation in directions respectively	δ_{ij}	Components of unit invariant tensor

SUBSCRIPTS

i, j	Indices
L	Lower skin
U	Upper skin
W	Webs and ribs
x, y, z	Space coordinates

TABLE 2

AEROELASTIC SYMBOLS

E	Young's modulus
h	Vertical translation, bending frequency effects
I	Moment of inertia of wing segment about the elastic axis
m	Mass of wing segment per foot
t	Time
V	Flutter speed
ρ	Density
ω	Flutter frequency

Subscripts

F	Flutter effects	α	Torsional frequency effects

1.0 SOLID STRUCTURES

(1.1) *Stress Field in a Solid Body*

Thermoelastic equations. If the temperature at the point p of the body is increased by θ, deformation can be produced without any corresponding change of pressure. If forces are applied at the point in addition to the increase of temperature, the strain at the point consists of the thermal extension superimposed upon the strain connected with the stress. The thermal expansion, which all linear elements of the body undergo, is $\propto \theta$, where $\propto$ is the coefficient of thermal expansion of the body material and θ is the temperature increase.

The stress-strain relation for a body strained by change of temperature, in addition to body forces, then, is given as follows:

$$
\begin{aligned}
\epsilon_{xx} &= \frac{1}{E}\,[\sigma_x - \nu\,(\sigma_y + \sigma_z)] + \alpha\theta & \gamma_{xy} &= \frac{2\,(1+\nu)}{E}\,\tau_{xy} \\
\epsilon_{yy} &= \frac{1}{E}\,[\sigma_y - \nu\,(\sigma_z + \sigma_x)] + \alpha\theta & \gamma_{yz} &= \frac{2\,(1+\nu)}{E}\,\tau_{yz} \qquad (1.1) \\
\epsilon_{zz} &= \frac{1}{E}\,[\sigma_z - \nu\,(\sigma_x + \sigma_y)] + \alpha\theta & \gamma_{zx} &= \frac{2\,(1+\nu)}{E}\,\tau_{zx}
\end{aligned}
$$

General equations of equilibrium and compatibility. The general equations of equilibrium of the surface stresses with the external forces on the surface can be written as:

$$
\begin{aligned}
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} &= -F_x \\
\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} &= -F_y \qquad (1.2) \\
\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} &= -F_z
\end{aligned}
$$

where F_x, F_y, and F_z are components of external force.

The conditions of compatibility are:

$$
\begin{aligned}
\frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} &= \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} & \frac{\partial}{\partial x}\left(-\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z}\right) &= 2\,\frac{\partial^2 \epsilon_x}{\partial y \partial z} \\
\frac{\partial^2 \epsilon_y}{\partial z^2} + \frac{\partial^2 \epsilon_z}{\partial y^2} &= \frac{\partial^2 \gamma_{yz}}{\partial y \partial z} & \frac{\partial}{\partial y}\left(\frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{yz}}{\partial y} + \frac{\partial \gamma_{zy}}{\partial z}\right) &= 2\,\frac{\partial^2 \epsilon_y}{\partial x \partial z} \qquad (1.3) \\
\frac{\partial^2 \epsilon_z}{\partial x^2} + \frac{\partial^2 \epsilon_x}{\partial z^2} &= \frac{\partial^2 \gamma_{xz}}{\partial x \partial z} & \frac{\partial}{\partial z}\left(\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z}\right) &= 2\,\frac{\partial^2 \epsilon_z}{\partial x \partial y}
\end{aligned}
$$

Boundary conditions. The total on the surface of the body consists of normal aerodynamic pressure $-P\delta ij$ and the viscosity stress of the gas (aerodynamic friction drag) $\mu\,(\partial v_j/\partial x_i + \partial v_i/\partial x_j)$. The boundary conditions are, therefore,

$$\sigma_{ij} - \mu\left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j}\right) + P\delta ij = 0 \tag{1.4}$$

In addition, boundary conditions regarding the supporting structure of the prototype should be duplicated in the model.

(1.2) *Elastic Similarity Conditions*

On the basis of EQUATIONS 1.1, 1.2, and 1.3, we can write the following similarity equations for the stress field in a body produced by external forces and temperature:

$$\frac{\sigma_{ij}}{E} = f\left[\frac{(\sigma_{ij})_F}{p}, \frac{\tau_{ij}}{E}, \frac{e_i}{L}, \frac{\theta_s}{\theta_0}, \alpha\theta_0, \nu, \frac{x_i}{L}\right] \tag{1.5}$$

where $e_i = u$, v, and w. Similarity equation 1.5 gives the following similarity conditions:

$$N_{x_i} = N_{e_i} = N_L \tag{1.6}$$

$$N_{\sigma_{ij}} = N_{(\sigma_{ij})_F} = N_{\tau_{ij}} = N_E = N_P \tag{1.7}$$

$$N_\alpha N_\theta = 1 \tag{1.8}$$

$$N_\nu = 1 \tag{1.9}$$

2.0 WING STRUCTURES

2.1 *Mean Deflection of Wing Structures*

Consider an element of the wing structure (FIGURE 2.1) that is subjected to lateral forces and forces acting in the middle plane of the skin. Tangential forces are denoted by N_x, N_y, N_{xy}, and N_{yx}, and for the body forces or tangential forces per unit area of the median surface we have X and Y acting along the x and y directions respectively.

By projecting these forces on the x and y axes we obtain the following equilibrium equations:

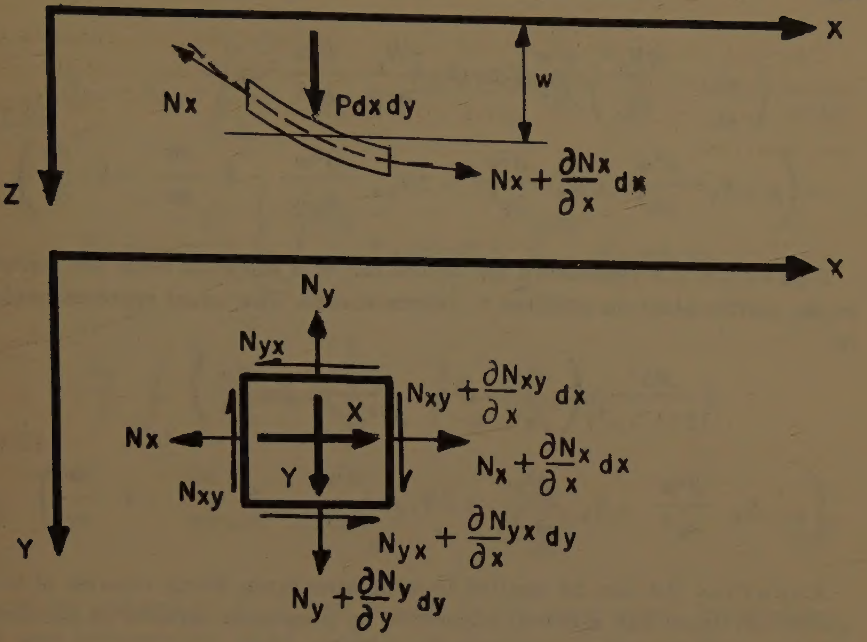

FIGURE 2.1. Simplified wing section element.

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = -X \tag{2.1}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = -Y$$

The equation of equilibrium is formed by equating the summation of the normal and shearing force projections on the z axis to the load on the element. We can write that

$$\left(p + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y}\right) + \frac{\partial w}{\partial x}\left(\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y}\right)$$

$$+ \frac{\partial w}{\partial y}\left(\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x}\right) + \frac{\partial N_x}{\partial x} \frac{\partial^2 w}{\partial x^2} d_x + \frac{\partial N_y}{\partial y} \frac{\partial^2 w}{\partial y^2} dy = 0 \tag{2.2}$$

Substitution of EQUATION 2.1 gives

$$\frac{\partial N_x}{\partial x}\,\frac{\partial^2 w}{\partial x^2}\,dx + \frac{\partial N_y}{\partial y}\,\frac{\partial^2 w}{\partial y^2}\,dy = -\left(p + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} - X \frac{\partial w}{\partial x} - Y \frac{\partial w}{\partial y}\right) \tag{2.3}$$

EQUATION 2.3 represents the deflection of a plate, if there are forces in the middle plane in addition to lateral forces. The usual representation is

$$\frac{Eh^3}{12(1-\nu^2)}\left(\frac{\partial^4 w}{\partial x^4} + 2\,\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) = \left(p + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} - X \frac{\partial w}{\partial x} - Y \frac{\partial w}{\partial y}\right) \tag{2.4}$$

EQUATION 2.4 can be applied to wing structures which consist of two plates (an upper and a lower) connected by a common member or members with a spacing of δ as shown in FIGURE 2.2.

We define N_x, N_y, $N_{yx} = N_{xy}$ and N_θ as follows:

$$N_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_z dz = \frac{E_U d_U + E_L d_L}{1-\nu^2}\left(\frac{\partial u}{\partial x} + \nu \frac{\partial v}{\partial y}\right) + \frac{E_w d_w h}{\delta}\,\frac{\partial u}{\partial x} \tag{2.5}$$

$$N_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y d_{vz} = \frac{E_U d_U + E_L d_L}{1-\nu^2}\left(\frac{\partial v}{\partial y} + \nu \frac{\partial u}{\partial x}\right) + \frac{E_w d_w h}{\delta}\,\frac{\partial v}{\partial y} \tag{2.6}$$

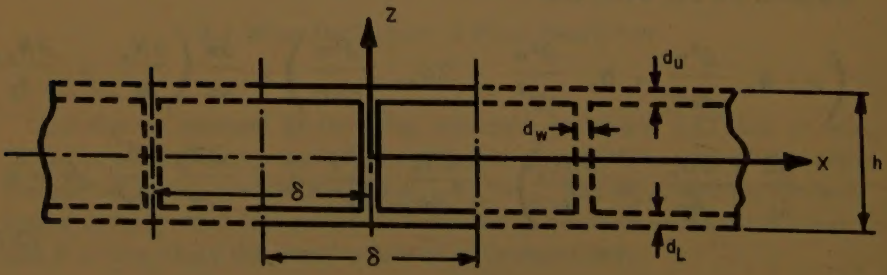

FIGURE 2.2. Simplified wing section.

$$N_{xy} = N_{yx} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy}\,dx = \frac{E_U d_U + E_L d_L}{1+\nu}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \tag{2.7}$$

$$N_\theta = -\frac{1}{1-\nu}\left\{\int_0^{d_U} E_U \propto \theta_U dz + \int_0^{d_L} E_L \propto \theta_L dz + \frac{d_w}{\delta}\int_{-\frac{h}{2}}^{\frac{h}{2}} E_w a\theta_w dz\right\} + \frac{E_\theta}{1-\nu}\left(d_U + d_L + \frac{d_w}{\delta}\right) \tag{2.8}$$

$$\text{and} \quad X = \frac{\partial N_\theta}{\partial x}, \quad Y = \frac{\partial N_\theta}{\partial y} \tag{2.9}$$

where the subscripts U, L, and W refer to the upper plate, lower plate, and web respectively. Also, u and v are components of deflection in the u and v directions respectively.

2.2 *Conditions for Similarity in Mean Deflection*

Since $X = \partial N_\theta / \partial x$ and $Y = \partial N_\theta / \partial y$ we can deduce from EQUATIONS 2.2 the following similarity conditions:

$$N_N = N_\theta \quad \text{with} \quad N_W = N_L \tag{2.10}$$

EQUATION 2.4 gives

$$\frac{N_E N_{dU}^3 N_W}{N_\delta^4 N_\nu^2} = \frac{N_N N_W}{N_\delta^2} = \frac{N_\theta N_W}{N_\delta^2} \quad \text{or} \quad N_N = \frac{N_E N_{dU}}{N_\nu^2} \tag{2.11}$$

$$\frac{N_E N_{dU}^3 N_W}{N_\delta^4 N_\nu^2} = \frac{N_E N_{dU} N_W}{N_\delta^2 N_\nu^2} \quad \text{or} \quad N_{dU} = N_\delta \tag{2.12}$$

EQUATION 2.7 gives

$$N_N = \frac{N_E N_{d_U} N_u}{N_\nu N_L} = \frac{N_E N_{d_U} N_v}{N_\nu N_L} \quad \text{or} \quad N_u = N_v \tag{2.13}$$

EQUATION 2.8 gives

$$N_\theta = \frac{N_E N_\alpha N_\theta N_{dU}}{N_\nu} = \frac{N_E N_\alpha N_\theta N_{dW} N_L}{N_\delta N_L} \quad \text{or} \quad N_{dW} = \frac{N_{dU} N_\delta}{N_L} \tag{2.14}$$

The similarity conditions given by EQUATIONS 2.10, 2.11, and 2.14 can be summarized

$$\frac{N_E N_{dz}}{N_\nu^{\,2}} = \frac{N_E N_\alpha N_\theta N_{dz}}{N_\nu} \quad \text{or} \quad N_\alpha N_\theta = \frac{1}{N_\nu} \tag{2.15}$$

2.3 *Local Skin Buckling of Wing Structures*

In the consideration of local buckling, we assume that there are no body forces or tangential forces acting in the x and y directions at the faces of the plate; that is, X and Y are zero in EQUATION 2.1. We then have, for the equilibrium equation,

$$\begin{aligned} \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} &= 0 \\ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} &= 0 \end{aligned} \tag{2.16}$$

EQUATION 2.4 simplifies to

$$\begin{aligned} \frac{Eh^3}{12(1-\nu^2)} \left(\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) &= \\ p + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} &+ 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} \end{aligned} \tag{2.17}$$

where now

$$N_x = \int_0^{d_U} \sigma_x d_U = \frac{E_U d_U}{1-\nu^2} \left[\left(\frac{\partial u}{\partial x} + \nu \frac{\partial v}{\partial y} \right) - \frac{h}{2} \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) \right] \tag{2.18}$$

$$N_y = \int_0^{d_U} \sigma_y d_U = \frac{E_U d_U}{1-\nu^2} \left[\left(\frac{\partial v}{\partial x} + \nu \frac{\partial u}{\partial y} \right) - \frac{h}{2} \left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right) \right] \tag{2.19}$$

$$N_{xy} = N_{yx} = \int_0^{d_U} \tau_{xy} d_U' = \frac{E_U d_U}{1+\nu} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \tag{2.20}$$

or for the lower plate, we use d_L instead of d_U.

2.4 *Similarity Conditions for Local Buckling*

EQUATION 2.17 gives

$$\frac{N_E N_{dU}^3 N_w}{N_\delta^4 N_\nu^2} = \frac{N_N N_w}{N_\delta^2} = \frac{N_\delta N_{dU} N_w}{N_\delta^2 N_\nu^2} \quad \text{or} \quad N_{d_u} = N_\delta \tag{2.21}$$

$$\frac{N_E N_{dU}^3 N_w}{N_\delta^4 N_\nu^2} = p \quad \text{or} \quad N_w = \frac{N_{du}^2}{N_L} \tag{2.22}$$

From EQUATION 2.18 we have

$$N_N = \frac{N_E N_{d_U} N_u}{N_\nu^2 N_L} = \frac{N_E N_{d_U} N_L N_w}{N_\nu^2 N_L} \quad \text{or} \quad N_u = N_v \tag{2.23}$$

3.0 SUMMARY AND APPLICATION OF SIMILARITY PARAMETERS

3.1 *Summary of Similarity Parameters*

It follows that to design a model is to simulate the structural behavior of a body in the presence of elevated temperatures except for scale factors which are approximately one. As illustrated by similarity EQUATIONS 1.7 and 1.8, true simulation of the deformation of thermal origin ($N_{\epsilon_\theta} = N_\alpha N_\theta$) and that of mechanical origin ($N_{\epsilon_p} = N_p/N_E$) and can be only achieved with different scale factors. Of course, when one of the deformation effects (thermal or mechanical) is negligible compared with the other, the similarity in the remaining large deformation effect can be obtained directly through its similarity laws.

For completeness, TABLE 3.1 summarizes the similarity laws for simulation of aerodynamic (mechanical), elastic, inertial, and heat transfer effects. The complete derivation of the laws shown therein is given in Scipio and Teng, 1954 (Part I).

3.2 *Application of Similarity Parameters*

FIGURES 3.1 and 3.2 show the transient aerodynamic heating phase and those regions of negligibility of either the thermal or mechanical effects. FIGURES 3.1 and 3.2 are applicable to solid and wing-type structures, respectively.

TABLE 3.1

SIMILARITY PARAMETERS FOR WIND TUNNEL TESTING OF AIRCRAFT STRUCTURES

Item	*Phenomena investigated*			
	Aerodynamic effects	*Thermal effects*	*Elastic effects*	*Aeroelastic effects*
Information obtainable	(1) Coefficients of forces and moments (C_L, C_M, etc.) (2) Pressure distribution around the model	(1) Surface coefficients of heat transfer and temperature recovery factors, etc. (2) Temperature distributions (surface and internal)	(1) Deflections (vertical and angular) (2) Strain distributions	(1) Vibration Modes (2) Critical flutter speed (3) Amplitudes of vibration (4) General interaction
General similarity criteria	Same Mach number Same Reynolds number Approx. same Prandtl number $N_\rho P_L = N_\theta^{1/2}$	$N_{K_s} = \frac{N_k N_\theta N_{d_s}}{N_{\theta_s} N_L}$ $N_{t_s} = \frac{N_{d_s}^2}{N_{K_s}}$ $N_{t_w} = \frac{N_L^2}{N_{K_w}}$	$N_\alpha N_\theta = \frac{1}{N_\nu}$ $N_{d_s} = \frac{N_\rho N_L N_\nu^2}{N_E}$ $N_{d_w} = \frac{N_{d_s} N_\delta}{N_L}$ $N_\delta = N_{d_s}$ $N_u = N_v = N_\delta$ $N_w = \frac{N_{d_s}^2}{N_L}$	$N_m = N_\rho N_L^2$ $N_{I_\alpha} = N_m N_L^3$ $N_{EI} = N_m N_L^4/N_t^2$ $N_{\omega_F} = [N_{EI}/N_m N_L^4]^{1/2}$ $N_v = N_w N_L$ $N_t = N_L/N_v$ $N_h = N_L$
Special similarity criteria*	Same Mach number Same Reynolds number Approx. same Prandtl number with $N_\theta = 1$ $N_\rho = \frac{1}{N_L}$ $N_p = \frac{1}{N_L}$	$N_\theta = 1$ $N_{K_s} = 1$ $N_{t_s} = \frac{N_L^2}{N_{K_s}}$ $N_{t_w} = \frac{N_L^2}{N_{K_w}}$	If the mechanical and thermal effects are of the same order of magnitude and in addition, the model and prototype materials are identical with unscaled temperatures, similarity cannot be achieved.**	$N_m = N_L$ $N_{I_\alpha} = N_L^3$ $N_{EI} = N_L^3$ $N_{\omega_F} = [N_{EI}/N_m N_L^4]^{1/2}$ $N_t = N_L$ $N_h = N_L$

* Model and prototype are constructed of identical materials, and temperature unscaled

**See FIGURE 3.1.

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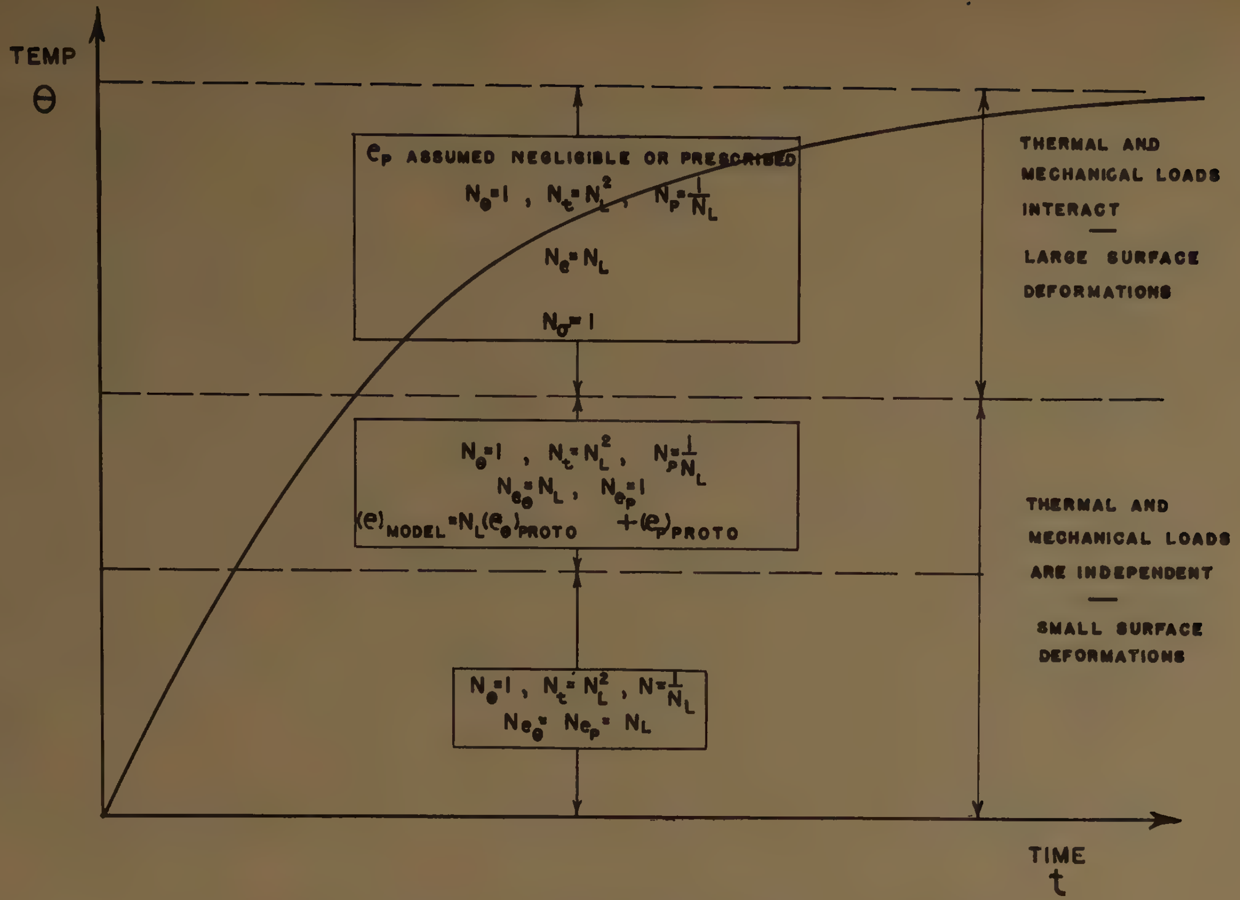
TEMP
Θ
e_p ASSUMED NEGLIGIBLE OR PRESCRIBED
$N_\theta = 1$, $N_t = N_L^2$, $N_p = \frac{1}{N_L}$
$N_e = N_L$
$N_\sigma = 1$
$N_\theta = 1$, $N_t = N_L^2$, $N_\rho = \frac{1}{N_L}$
$N_{e_\theta} = N_L$, $N_{e_p} = 1$
$(e)_{MODEL} = N_L (e_\theta)_{PROTO} + (e_p)_{PROTO}$
$N_\theta = 1$, $N_t = N_L^2$, $N = \frac{1}{N_L}$
$N_{e_\theta} = N_{e_p} = N_L$
THERMAL AND
MECHANICAL LOADS
INTERACT
—
LARGE SURFACE
DEFORMATIONS
THERMAL AND
MECHANICAL LOADS
ARE INDEPENDENT
—
SMALL SURFACE
DEFORMATIONS
TIME
t

FIGURE 3.1. Transient similarity regions of solid structures.

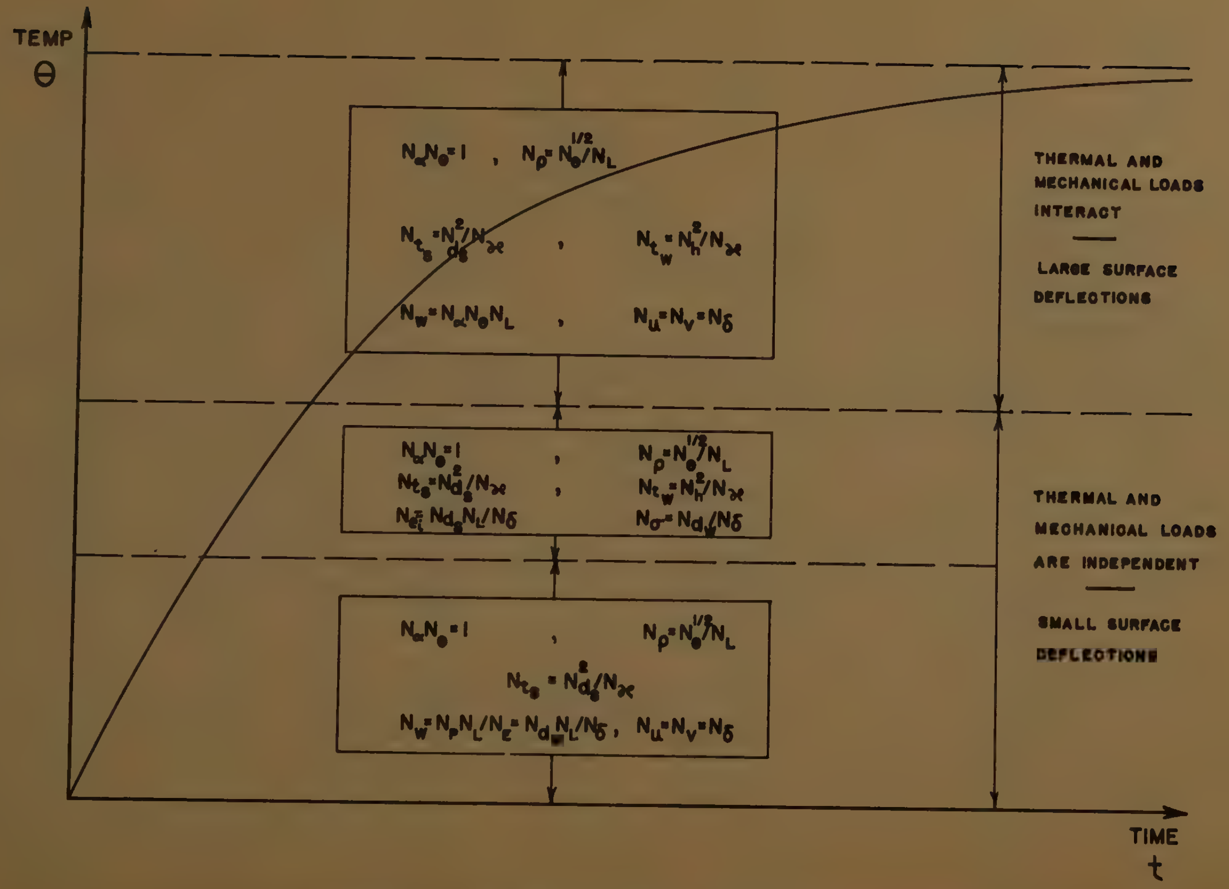
TEMP
Θ
$N_\alpha N_\Theta = 1$, $N_\rho = N_\Theta^{1/2}/N_L$
$N_{t_s} = N_{d_s}^2/N_\varkappa$, $N_{t_w} = N_h^2/N_\varkappa$
$N_w = N_\alpha N_\Theta N_L$, $N_u = N_v = N_\delta$
$N_\alpha N_\Theta = 1$, $N_\rho = N_\Theta^{1/2}/N_L$
$N_{t_s} = N_{d_s}^2/N_\varkappa$, $N_{t_w} = N_h^2/N_\varkappa$
$N_{\epsilon_i} = N_{d_s} N_L/N_\delta$, $N_\sigma = N_{d_w}/N_\delta$
$N_\alpha N_\Theta = 1$, $N_\rho = N_\Theta^{1/2}/N_L$
$N_{t_s} = N_{d_s}^2/N_\varkappa$
$N_w = N_\rho N_L/N_\epsilon = N_{d_w} N_L/N_\delta$, $N_u = N_v = N_\delta$
THERMAL AND MECHANICAL LOADS INTERACT
LARGE SURFACE DEFLECTIONS
THERMAL AND MECHANICAL LOADS ARE INDEPENDENT
SMALL SURFACE DEFLECTIONS
TIME
t

FIGURE 3.2. Transient similarity regions of wing structures.

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